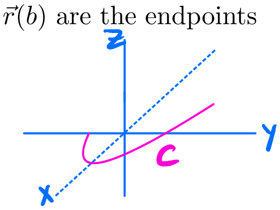
MATH 112A Review: Line Integrals, Surface Integrals, Parametrization of Curves

Facts to Know:

(Line integral of a scalar field) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a scalar field and let $\vec{r}(t)$ be a bijective parametrization of a curve C in $\mathbb{R}^n$ with parameter $t \in [a, b]$ such that $\vec{r}(a)$ and $\vec{r}(b)$ are the endpoints of C. Then the line integral along C is

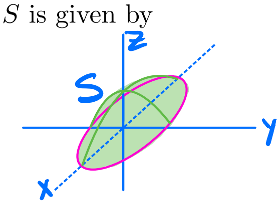


$$\int_C f(x_1, \ldots, x_n) ds =$$

(Line integral of a vector field) Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be a vector field and let $\vec{r}(t)$ be a bijective parametrization of a curve C in $\mathbb{R}^n$ with parameter $t \in [a, b]$ such that $\vec{r}(a)$ and $\vec{r}(b)$ are the endpoints of C. Then the line integral along C is in the direction of $\vec{r}$ is

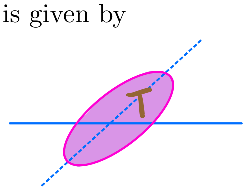
$$\int_C F(\vec{r}) \cdot d\vec{r} =$$

(Surface integral of a scalar field) Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a scalar field and let $\vec{r}(s, t)$ be a parametrization of a surface S in $\mathbb{R}^3$ with (s, t) vary in some region T in the plain. Then, the surface integral over S is given by



$$\iint_S f(x, y, z) dS =$$

(Surface integral of a vector field) Let $F : \mathbb{R}^3 \to \mathbb{R}$ be a scalar field and let $\vec{r}(s, t)$ be a parametrization of a surface S in $\mathbb{R}^3$ with (s, t) vary in some region T in the plain. Then, the surface integral over S is given by



$$\iint_S F \cdot d\vec{S} =$$

Examples:

1. Let $F(x, y) = (P(x, y), Q(x, y))$ and let $\vec{r}(t) = (x(t), y(t))$ be the parametrization of C. What is the line integral?

2. Let $F(x, y) = (y, -x)$ and consider the parametrization $\vec{r}(t) = (\cos t, \sin t)$ for $t \in [0, 2\pi]$ of the unit circle C with counterclockwise orientation. Compute

$$\int_C F(\vec{r}) \cdot \vec{r}\,'(t) dt$$